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A TREE-BY-TREE MEASURE OF SITE UTILIZATION FOR GRAND FIR RELATED TO STAND DENSITY INDEX

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ABSTRACT

The usefulness of stand density index (SDI) has been limited by lack of a way to partition its nonlinear expression into additive components to describe the relative stocking of a stand by species or quality classes. In this paper, a linear equation is derived to permit such a partition.

A closely related expression for grand fir stocking is given that retains the merits of SDI, but is more nearly proportional to growth of individual trees within a stand. Hence, the partitioning of stocking into components will be more meaningful to the forest manager with this revised form.

One important use of a measure of stocking is to describe the relative composition of the growing stock according to species, quality, and size classes. However, interpretation of the measured composition depends on the units of measurement. In a related paper,[2] I have proposed that many aspects of stocking condition and control can be described in terms of greater management significance when stocking is measured in units proportional to site utilization. Site utilization can be measured from current growth data. However, when direct sampling of growth is not feasible, the silviculturist must fall back on some other way to measure stocking that approximates the distribution of growth among trees within the stand of interest. To be useful for describing composition, this alternative measure of stocking should be

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[2]Stage, Albert R. A growth definition for stocking: units, sampling, and interpretation. (In preparation.)

additive, tree by tree, so that the total stocking for the stand will be the sum of its parts. The purpose of this Note is to describe an additive stocking measure for grand fir that is approximately proportional to growth.

Stand Density Index as proposed by Reineke[3] is a useful measure of stocking that relates the logarithm of number of trees per acre to the logarithm of tree size. The merits of this measure in contrast to polynomial measures of stocking such as basal area, tree-area ratio, and crown competition factor have been demonstrated by David Bruce.[4] He showed that the polynomial measures, throughout the range of their usefulness, are closely proportional to Stand Density Index. Furthermore, he argues that because Stand Density Index can be extrapolated to describe the effect of density on growth in stands as young as nursery seedlings, it must be the more generally valid form to use as an index of relative stocking.

A disadvantage of Stand Density Index is that there has been no way to describe the contribution of various classes of trees in the stand to the total index for the stand. The purpose of this Note is to demonstrate how this calculation can be accomplished, and to present a modified version of Stand Density Index for grand fir (Abies grandis (Dougl.) Lindl.).

COMPUTING STAND DENSITY INDEX TREE BY TREE

Stand Density Index as computed by Reineke is essentially

$$\log \text{SDI} = \log N + k \log\left(\sum_{i=1}^{N} d_i^2/N\right)^{1/2} - k \qquad (1)$$

in which k is a constant equal to 1.605, N is number of trees per acre, and $(\Sigma d_i^2/N)^{1/2}$ is the diameter of the tree of average basal area. The index thus computed is stated in terms of number of trees per acre for a stand of 10 inches average d.b.h. In this form, the index is commonly used to compare densities from stand to stand.

Stand Density Index can be computed tree by tree within a stand. That is, the total SDI can be subdivided to represent the contribution to SDI of different classes of trees in the stand. The equation for SDI that is additive tree by tree is

$$\text{SDI} = \sum_{i=1}^{N} (a + bd_i^2) \qquad (2)$$

or:

$$\text{SDI} = Na + b\sum_{i=1}^{N} d_i^2$$

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[3]Reineke, L. H. Perfecting a stand-density index for even-aged forests. J. Agr. Res. 46(7):627-638. 1933.

[4]Bruce, David. Unpublished paper presented at Lodgepole Pine Conference, Bend, Oregon. 1965.

where a and b are coefficients depending on the average diameter of all trees, and d_i is the diameter of the i^{th} tree in the tally. The coefficients are determined from the diameter of the tree of average basal area by the following equations:

$$a = 10^{-k} \left(1 - \frac{k}{2}\right) \left(\Sigma d_i^2/N\right)^{k/2}$$
$$b = 10^{-k} \left(\frac{k}{2}\right) \left(\Sigma d_i^2/N\right)^{(k/2)-1} \qquad (3)$$

Thus, if we have two classes of trees in a stand, the stocking of each component can be computed from equation (2) above by summing the values of $a + bd^2$ for the trees in each class. In turn, the total Stand Density Index will be the sum of the partial SDI's in each class.

A proof that substituting the coefficients a and b as defined in equation (3) into equation (2) yields an expression equivalent to (1) is given in the appendix to this Note.

APPLICATION TO GRAND FIR SITE UTILIZATION

Two modifications of SDI are here proposed for application to grand fir stands. First, for stands of a given average d.b.h., the index will be rescaled so that an index of 100 corresponds to the number of trees needed to fully utilize the site. Second, the form of the index within a stand will be changed from the form in equation (2) to be proportional to diameter cubed.

The reason for this latter change is to make the contributions to the per-acre stocking more nearly proportional to the growth of the individual trees. In even-aged stands of grand fir, growth per tree has been shown to be proportional to diameter cubed.[5] If growth is the criterion for rating contribution to stocking, the equation (2) for SDI given above overrates the smaller trees and underrates the larger trees.

Through analysis of growth data from study plots in even-aged grand fir stands,[5] the following expression was found to estimate the minimum number of trees per acre required to fully utilize a site capable of growing grand fir:

$$\log N = 4.26820 - 1.76235 \log\left(\sum_{i=1}^{N} d^3/N\right)^{1/3} \qquad (4)$$

This equation is clearly related to equation (1) for SDI given above. The principal difference is in the measure of average tree size. The cube root of the average of the cubed diameters has been substituted for the square root of the average of the squared diameters.

To emphasize the contribution of Σd^3 in (4), consider this restatement of the same relation:

$$\log \Sigma d_i^3 = \log \left(\Sigma d_i^3/N\right)^{1-(k/3)} + 4.26820 \qquad (5)$$

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[5]Stage, Albert R. A study of growth of grand fir (Abies grandis (Dougl.) Lindl.) in relation to site quality and stocking. Ph.D. Diss. Univ. Michigan, Ann Arbor. 1966.

in which k = 1.76235. Taking antilogs of (5) leads to

$$\Sigma\, d_i^3 = 18544. \quad (\Sigma\, d_i^3/N)^{0.41255}$$

In this form, the right-hand side of this last expression gives the total sum of diameters cubed needed (per acre) to utilize the site completely at the stage of development given by $\Sigma\, d_i^3/N$, the mean of cubed diameters. Hence, the percent utilization of the site by trees can be expressed as

$$S = \frac{100\ \Sigma\, d_i^3}{18544.\ (\Sigma\, d_i^3/N)^{0.41255}}$$

or, simplifying:

$$S = \frac{\Sigma\, d_i^3}{185.44\ (\Sigma\, d_i^3/N)^{0.41255}}$$

Note that the denominator contains the average of cubed diameters over all trees in the stand. Table 1 lists the values of the denominator for stand diameters from 3 to 30 inches.

The sum in the numerator can then be separated into subtotals over those classes of trees in the stand for which stocking fractions are desired.

Obviously, it is very likely that stands will rate over 100 percent by this method, yet it is not possible to have the site more than fully utilized. The appropriate interpretation of a stocking index over 100, say, 125 percent, is that a fraction equal to 25/125 or 20 percent of the stocking could be removed, yet leave sufficient growing stock to fully utilize the site after sufficient time has passed to allow the remaining trees to adjust to the new level of stocking.

Site utilization of less than 100 percent could well be a desirable objective in young stands before any commercial thinning opportunities exist. In such stands, the value of more rapid diameter growth might well exceed the value of the unused site potential.

Example

Site utilization calculations for a sample tally from a fifth-acre plot in a young grand fir stand are illustrated in table 2. In this example, the site is apparently 99 percent utilized, about 48 percent of the growth capacity of the site being used by desirable trees.

Table 1.--Values useful in estimating stocking for grand fir.

Diameter	d^3	$185.44(\Sigma d_i^3/N)^{0.41255}$
3	27	722
4	64	1,031
5	125	1,359
6	216	1,703
7	343	2,061
8	512	2,432
9	729	2,813
10	1,000	3,205
11	1,331	3,606
12	1,728	4,016
13	2,197	4,435
14	2,744	4,861
15	3,375	5,294
16	4,096	5,734
17	4,913	6,181
18	5,832	6,634
19	6,859	7,093
20	8,000	7,558
21	9,291	8,029
22	10,648	8,504
23	12,167	8,986
24	13,824	9,471
25	15,625	9,962
26	17,576	10,458
27	19,683	10,958
28	21,952	11,462
29	24,389	11,971
30	27,000	12,484

Table 2.--Sample stocking calculations for a fifth-acre plot tally.

D.b.h.	Number of trees				Σd_i^3			
	Desirable	Acceptable	Cull	Total	Desirable	Acceptable	Cull	Total
3			11	11			297	297
4		15	9	24		960	576	1,536
5	13	20		33	1,625	2,500		4,125
6	30	40	4	74	6,480	8,640	864	15,984
7	18	5		23	6,174	1,715		7,889
8	2		2	4	1,024		1,024	2,048
Total	63	80	26	169	15,303	13,815	2,761	31,879

$\Sigma d_i^3/N = 31879/169 = 188.6$

$185.44\ (\Sigma d_i^3/N)^{0.41255} = 1611$[1]

$\text{Total } S = \frac{(31879)\ (5)}{1611} = 98.9\%$

$\text{Desirable } S = \frac{(15303)\ (5)}{1611} = 47.5\%$

[1]Linear interpolation for $d^3 = 188.6$ in table 1 would have given $185.44\ (\Sigma d_i^3/N)^{0.41255}$ to be approximately 1599 instead of 1611. The effect of this difference on stocking interpretations would be negligible.

APPENDIX

PROOF OF TREE-BY-TREE EXPRESSION OF SDI

Equation (2), with coefficients a and b defined by equation (3) becomes

$$SDI = \sum_{i=1}^{N} (a + bd_i^2) = Na + b \sum_{i=1}^{N} d_i^2$$

$$= 10^{-k} (1 - k/2) \left(\sum_{i=1}^{N} d^2/N \right)^{k/2} N$$

$$+ 10^{-k} (k/2) \left(\sum_{i=1}^{N} d^2/N \right)^{(k/2)-1} \left(\sum_{i=1}^{N} d_i^2 \right)$$

Hence

$$SDI = 10^{-k} \left[(1 - k/2) \left(\sum_{i=1}^{N} d^2 \right)^{k/2} N^{1 - k/2} + (k/2) (N^{1 - k/2}) \left(\sum_{i=1}^{N} d_i^2 \right)^{k/2} \right]$$

$$= 10^{-k} \left(\sum_{i=1}^{N} d_i \right)^{k/2} N^{1 - k/2}$$

Taking logarithms to base 10

$$\log SDI = -k + k/2 \log \left(\sum_{i=1}^{N} d_i^2 \right) + (1 - k/2) \log N$$

$$\log SDI = k/2 \left(\log \sum_{i=1}^{N} d_i^2 - \log N \right) + \log N - k$$

$$\log SDI = \log N + k \log \left(\Sigma d_i^2/N \right)^{1/2} - k$$

This last equation is identical with (1) which was to be proven.

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